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EXTRAPOLATING SU(3) BREAKING EFFECTS FROM D TO B DECAYS

Michael Gronau
Physics Department, Technion – Israel Institute of Technology
32000 Haifa, Israel

and

Dan Pirjol
Department of Physics, University of California at San Diego
La Jolla, CA 92093

ABSTRACT

We consider two SU(3) breaking parameters, $R_1(m_B)$ and $R_2(m_B)$, appearing in a relation between $B^+ \to K\pi$ and $B^+ \to \pi\pi$ amplitudes, which plays an important role in determining the weak phase γ. We identify an isospin-related quantity $R_2(m_D)$ measured in D decays, exhibiting large SU(3) breaking which is likely due to nonfactorizable effects. With a cautious remark about possible nonfactorizable SU(3) breaking in B decays, we proceed to calculate factorizable SU(3) breaking corrections. Applying heavy quark symmetry to semileptonic D and B decay form factors, we find that SU(3) breaking in $R_2(m_B)/R_1(m_B)$ may be significantly larger than estimated from certain model calculations of form factors.

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Weak nonleptonic decays of B mesons provide an important source of information about the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Flavor SU(3) symmetry of strong interactions plays an essential role in some of the methods proposed to determine the weak phases [1]. First order SU(3) breaking effects in hadronioc B decays may be parametrized in a completely general way in terms of several unknown parameters [2] some of which can be determined from experiments. In certain hadronic amplitudes, such as in $B \to \bar{D}\pi$, experimental evidence exists for factorization in terms of products of two current matrix elements [3]. In these cases, the corresponding SU(3) breaking parameters are given by ratios of K and π decay constants and ratios of B/B_s to D/D_s form factors. In decays to two charmless pseudoscalar mesons, which are useful for weak phase determinations [4], experimental evidence for factorization of hadronic matrix elements is still lacking. It was argued recently [5] that within QCD nonfactorizable corrections due to hard gluon exchange are calculable and those which are due to soft exchanges are suppressed by $\Lambda_{\rm QCD}/m_b$ in a heavy quark expansion. Actual calculations of these corrections, controlling the former in a model-independent manner and showing that the latter are indeed small, are both desirable and challenging. Furthermore, in order to treat SU(3) breaking within the factorization approximation, one still needs the values of certain ratios of unmeasured form factors, for which one oftens relies on theoretical models.

The purpose of this Letter is to learn about SU(3) breaking in B decays from the corresponding measured effects in D decays. SU(3) breaking does not necessarily decrease monotonously with the decaying heavy quark mass. We will try to address the two relevant questions, of factorizable and nonfactorizable SU(3) violating corrections to hadronic decays, and of SU(3) breaking in semileptonic form factors which are used in the factorization approximation.

In general, soft final state interactions which spoil factorization are expected to affect D and B decays differently. It was often argued [6], and it has recently been shown by an actual calculation [7], that D decay amplitudes involve large contributions from nearby light $q\bar{q}$ resonances which induce large SU(3) breaking effects. Such effects are not expected in B decays. To avoid resonance effects, and thus study D and B decays on common grounds, we will consider only decays to "exotic" final states involving $\pi\pi$ in $I = 2$ and $K\pi$ in $I = 3/2$.

We will find very large SU(3) breaking in hadronic D decays, in the absence of resonant terms, implying in the most likely scenario large nonfactorizable corrections. This should serve as a warning for what may be the case also in B decays. In the factorization approximation, we then proceed to calculate SU(3) breaking in hadronic B decays, where B meson form factors are obtained from those measured for D by applying a heavy quark symmetry scaling law. Our result will be compared with a model-dependent calculation.

As our test case, we consider an SU(3) relation between the isospin $I = 3/2$ amplitude in $B \to K\pi$ and the $I = 2$ amplitude in $B \to \pi\pi$ [8, 9]

$$\begin{aligned} A(B^+ \to K^0\pi^+) &+ \sqrt{2}A(B^+ \to K^+\pi^0) = \sqrt{2}\tan\theta_c(R_1 - \delta_+ e^{-i\gamma}R_2)A(B^+ \to \pi^0\pi^+) \ , \\ \delta_+ &\equiv -[3/(2\lambda|V_{ub}/V_{cb}|)][(c_9 + c_{10})/(c_1 + c_2)] = 0.66 \pm 0.15 \ . \end{aligned} \tag{1}$$

This SU(3) relation generalizes a triangle relation proposed in [10] by including, in addition to the current-current ("tree") contributions, also the effects of dominant electroweak penguin (EWP) amplitudes given by the second term on the right-hand-side. Eq. (1) and its charge-conjugate were proposed as a way for determining the weak phase $\gamma \equiv {\rm Arg}V^*_{ub}$. Other suggestions for using $B \to K\pi$ decays to study γ were discussed in [11].

The coefficients $R_{1,2}$ in Eq. (1) parametrize SU(3) breaking effects and are in general complex numbers. In the SU(3) limit they are both equal to 1. Knowledge of the precise values of R_1 and R_2/R_1, in the presence of SU(3) breaking, is crucial for an accurate determination of γ [8, 12, 13]. Using the factorization approximation, it is customary to apply the value $R_1 \simeq f_K/f_\pi = 1.22$ to the tree part. SU(3) breaking corrections to the EWP-to-tree ratio R_2/R_1 were estimated in the generalized factorization approximation, assuming a certain model-dependent value for the ratio of B to K and B to π form factors, and were found to amount to a few percent [8, 9]. Numerically, this follows from an accidental cancellation between the contributions of the color-allowed and color-suppressed amplitudes. In addition, nonfactorizable SU(3) breaking corrections can in principle be significant [12].

Our main concern will be the SU(3) breaking parameter R_2. We will show that there exists a corresponding quantity $R_2(m_D)$, which has already been measured in D decays exhibiting large SU(3) breaking. It will be argued that this effect is likely due to nonfactorizable corrections. It is not obvious why such effects should be much suppressed in B decays. With this cautious remark, we nevertheless assume factorization in order to calculate $R_1(m_B)$ and $R_2(m_B)$ in this approximation. Using heavy quark symmetry to extrapolate form factors from measured semileptonic D decays to B decays, we calculate factorizable SU(3) breaking in $R_2(m_B)/R_1(m_B)$ and compare with estimates based on certain models for form factors.

For completeness, and in order to define R_1 and R_2 in broken SU(3) and to prove Eq. (1), we start by quickly reviewing the SU(3) structure of the amplitudes entering Eq. (1). The tree and electroweak penguin four-quark operators describing charmless decays transform under flavor SU(3) as a sum of $\overline{\mathbf{3}}$, $\mathbf{6}$ and $\overline{\mathbf{15}}$ [14]

$$
\begin{aligned}
&\mathcal{H}_T^{\Delta S=1} + \mathcal{H}_T^{\Delta S=0} + \mathcal{H}_{EWP}^{\Delta S=1} = \\
&\quad \frac{G_F}{\sqrt{2}}\lambda_u^{(s)}[\frac{1}{2}(c_1-c_2)(-\bar{\mathbf{3}}^{(a)}_{I=0} - \mathbf{6}_{I=1}) + \frac{1}{2}(c_1+c_2)(-\overline{\mathbf{15}}_{I=1} - \frac{1}{\sqrt{2}}\overline{\mathbf{15}}_{I=0} + \frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})] \\
&+ \frac{G_F}{\sqrt{2}}\lambda_u^{(d)}[\frac{1}{2}(c_1-c_2)(\mathbf{6}_{I=\frac{1}{2}} - \bar{\mathbf{3}}^{(a)}_{I=\frac{1}{2}}) + \frac{1}{2}(c_1+c_2)(-\frac{2}{\sqrt{3}}\overline{\mathbf{15}}_{I=\frac{3}{2}} - \frac{1}{\sqrt{6}}\overline{\mathbf{15}}_{I=\frac{1}{2}} + \frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=\frac{1}{2}})] \\
&- \frac{G_F}{\sqrt{2}}\frac{\lambda_t^{(s)}}{2}\left(\frac{c_9-c_{10}}{2}(3\cdot\mathbf{6}_{I=1} + \bar{\mathbf{3}}^{(a)}_{I=0}) + \frac{c_9+c_{10}}{2}(-3\cdot\overline{\mathbf{15}}_{I=1} - \frac{3}{\sqrt{2}}\overline{\mathbf{15}}_{I=0} - \frac{1}{\sqrt{2}}\bar{\mathbf{3}}^{(s)}_{I=0})\right) ,
\end{aligned}
\tag{2}
$$

where $\lambda_q^{(q')} = V_{qb}^* V_{qq'}$. The explicit expressions of the four-quark operators appearing in the Hamiltonian can be found in [14].

The left-hand-side of Eq. (1) receives only contributions from the $\Delta S = 1,\ I = 1$ terms in the weak Hamiltonian, which transform as $\mathbf{6}$ and $\overline{\mathbf{15}}$, (QCD penguin operators are pure $I = 0$ and do not contribute)

$$
\begin{aligned}
&A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0) = \\
&\quad \lambda_u^{(s)}(C_{\overline{15}_{I=1}} + C_{6_{I=1}}) + \lambda_t^{(s)}\left(-\frac{3}{2}\frac{c_9+c_{10}}{c_1+c_2}C_{\overline{15}_{I=1}} + \frac{3}{2}\frac{c_9-c_{10}}{c_1-c_2}C_{6_{I=1}}\right) .
\end{aligned}
\tag{3}
$$

Here

$$
C_{\overline{15}_{I=1}}(m_B) = \frac{G_F}{\sqrt{2}}\frac{1}{2}(c_1+c_2)(\langle K^0\pi^+| - \overline{\mathbf{15}}_{I=1}|B^+\rangle + \sqrt{2}\langle K^+\pi^0| - \overline{\mathbf{15}}_{I=1}|B^+\rangle) \tag{4}
$$

and

$$C_{6_{I=1}}(m_B) = \frac{G_F}{\sqrt{2}}\frac{1}{2}(c_1 - c_2)(\langle K^0\pi^+| - \mathbf{6}_{I=1}|B^+\rangle + \sqrt{2}\langle K^+\pi^0| - \mathbf{6}_{I=1}|B^+\rangle) \qquad (5)$$

are hadronic matrix elements of operators transforming as $\overline{\mathbf{15}}$ and $\mathbf{6}$. Using the approximate equality

$$\frac{c_9 + c_{10}}{c_1 + c_2} \approx \frac{c_9 - c_{10}}{c_1 - c_2} \approx -1.12\alpha \ , \qquad (6)$$

which holds to better than 3% [15], one finds

$$A(B^+ \to K^0\pi^+) + \sqrt{2}A(B^+ \to K^+\pi^0) = \lambda_u^{(s)}[(C_{\overline{15}_{I=1}} + C_{6_{I=1}}) - \delta_+ e^{-i\gamma}(C_{\overline{15}_{I=1}} - C_{6_{I=1}})] \ . \quad (7)$$

On the other hand, the amplitude on the right-hand-side of (1) is given by the matrix element of the $\Delta S = 0$, $I = 3/2$ term in the weak Hamiltonian, (we neglect a very small EWP contribution [14])

$$\sqrt{2}A(B^+ \to \pi^+\pi^0) = \lambda_u^{(d)} C_{\overline{15}_{I=3/2}} \ , \qquad (8)$$

where

$$C_{\overline{15}_{I=3/2}}(m_B) \quad = \quad \frac{G_F}{\sqrt{2}}(c_1 + c_2)\sqrt{\frac{2}{3}}\langle \pi^+\pi^0| - \overline{\mathbf{15}}_{I=3/2}|B^+\rangle \ . \qquad (9)$$

Taking the ratio of (3) and (8) reproduces the factor on the right-hand-side of (1) with

$$R_1(m_B) = \frac{C_{\overline{15}_{I=1}} + C_{6_{I=1}}}{C_{\overline{15}_{I=3/2}}} \ , \qquad R_2(m_B) = \frac{C_{\overline{15}_{I=1}} - C_{6_{I=1}}}{C_{\overline{15}_{I=3/2}}} \ . \qquad (10)$$

Both final states on the left-hand-side of (3) and (8) belong to a $\mathbf{27}$ multiplet of SU(3), such that the matrix elements of $\overline{\mathbf{15}}_{I=1}$ and $\overline{\mathbf{15}}_{I=3/2}$ are related in the SU(3) limit, $C_{\overline{15}_{I=1}} = C_{\overline{15}_{I=3/2}}$. (The different numerical factors defining these amplitudes in Eqs. (4) and (9) are related to the different isospins involved). Furthermore, the matrix element of $\mathbf{6}$ in (3) vanishes in the same limit, such that $R_1 = R_2 = 1$ in the SU(3) symmetric case. However, in broken SU(3) $C_{\overline{15}_{I=1}} \neq C_{\overline{15}_{I=3/2}}$ and $C_{6_{I=1}} \neq 0$, which causes both R_1 and R_2 to differ from unity.

Whereas $R_1(m_B)$ and $R_2(m_B)$ are purely theoretical quantities, which cannot be directly measured, we prove now that another SU(3) breaking parameter,

$$R_2(m_D) = -\frac{V_{us}}{V_{ud}}\frac{A(D^- \to K^0\pi^-)}{\sqrt{2}A(D^- \to \pi^-\pi^0)} \ , \qquad (11)$$

measured in D decays, is related to $R_2(m_B)$ by isospin in a fictitious heavy quark limit $m_c = m_b$.

The final states in the numerator and denominator of $R_2(m_D)$ have quantum numbers $|I = \frac{3}{2},\ I_3 = -\frac{3}{2}\rangle$ and $|I = 2,\ ,I_3 = -1\rangle$, respectively, and belong to the same isospin multiplets as the states $|K^0\pi^+\rangle + \sqrt{2}|K^+\pi^0\rangle$ and $|\pi^+\pi^0\rangle$ in (1). The initial states D^- and B^+ are related to each other by isospin in the (fictitious) limit of identical heavy quarks. The weak Hamiltonian responsible for the relevant $\bar{D}$ decays is

$$\begin{aligned}\mathcal{H}_W \quad &= \quad \frac{G_F}{\sqrt{2}}V_{ud}^*V_{cs}[\frac{1}{2}(c_1 - c_2)\sqrt{2}\ \mathbf{6}_{I=1} - \frac{1}{2}(c_1 + c_2)\sqrt{2}\ \overline{\mathbf{15}}_{I=1}] \qquad (12)\\ &+ \quad \frac{G_F}{\sqrt{2}}V_{us}^*V_{cs}[(c_1 + c_2)(\frac{1}{\sqrt{3}}\overline{\mathbf{15}}_{I=3/2} - \sqrt{\frac{2}{3}}\,\overline{\mathbf{15}}_{I=1/2}) + (c_1 - c_2)\,\mathbf{6}_{I=1/2}] \ ,\end{aligned}$$

where we neglect a small CP-violating contribution proportional to $\frac{1}{2}(V^*_{us}V_{cs}+V^*_{ud}V_{cd})=\mathcal{O}(\lambda^5)$ in the Cabibbo-suppressed part and very small contributions of penguin operators [16].

The $\Delta S=1$ ($\Delta S=0$) $I=1,\ I_3=-1$ ($I=\frac{3}{2},\ I_3=-\frac{1}{2}$) operators in (12) are the isospin partners of the $I=1,\ I_3=0$ ($I=\frac{3}{2},\ I_3=\frac{1}{2}$) operators in the B decay Hamiltonian (2). (This can also be shown in terms of their quark structure). Therefore, in the limit of identical heavy quarks, isospin symmetry of strong interactions relates the amplitudes for D^- decays in (11) to those in B^+ decays

$$\begin{aligned} A(D^-\to K^0\pi^-) &= V^*_{ud}V_{cs}(C_{\overline{15}_{I=1}}(m_D)-C_{6_{I=1}}(m_D))\ , && (13)\\ \sqrt{2}A(D^-\to\pi^-\pi^0) &= -V^*_{us}V_{cs}C_{\overline{15}_{I=3/2}}(m_D)\ . && (14)\end{aligned}$$

Taking the ratio of these amplitudes yields the SU(3) breaking parameter $R_2(m_D)$ given in Eq. (11).

The experimental value of the ratio of amplitudes (11) is [17]

$$|R_2(m_D)|=0.56\pm0.08\ . \qquad (15)$$

The large SU(3) breaking effect in $R_2(m_D)$ is somewhat surprising since the relevant final states are exotic, $I=\frac{3}{2}$ and 2, and thus receive no resonant contributions [7]. The large deviation of the ratio $|R_2(m_D)|$ from 1 raises the concern of a similar large SU(3) breaking effect in the B case. In view of this possibility, let us review previous attempts and difficulties in explaining the numerical value (15).

A common way of studying SU(3) breaking in hardonic D (and B) decays is by using the generalized factorization approach [18]. In this approach one finds

$$R_2(m_D)=\frac{a_2^{(DK\pi)}}{a_1^{(D\pi\pi)}+a_2^{(D\pi\pi)}}\frac{f_K}{f_\pi}\frac{F_0^{D\pi}(m_K^2)}{F_0^{D\pi}(m_\pi^2)}+\frac{a_1^{(DK\pi)}}{a_1^{(D\pi\pi)}+a_2^{(D\pi\pi)}}\frac{m_D^2-m_K^2}{m_D^2-m_\pi^2}\frac{F_0^{DK}(m_\pi^2)}{F_0^{D\pi}(m_\pi^2)}\ . \qquad (16)$$

The phenomenological parameters $a_{1,2}$, describing the external and internal W-emission amplitudes respectively, are related to corresponding Wilson coefficients through $a_{1,2}=c_{1,2}+\zeta c_{2,1}$. The parameter ζ is process- and scale-dependent and is determined from experiments. When fitting nonleptonic two-body $D\to K\pi$ decays, using $F_0^{DK}(m_\pi^2)=0.77$ [19] and $F_0^{D\pi}(m_\pi^2)=0.7$ [20], one obtains [18] $a_1^{(DK\pi)}=1.26$ and $a_2^{(DK\pi)}=-0.51$, corresponding to $\zeta(m_c)=0$. This is compatible with neglecting $1/N_c$ contributions in charm decays [21]. This fit neglects, however, resonance contributions in nonexotic channels which, when included in an appropriate way, modifies the extracted values of $a_{1,2}$ to become $a_1^{(DK\pi)}=1.06,\ a_2^{(DK\pi)}=-0.64$ [7].

An attempt was made [22] to explain the large SU(3) breaking in (15) by using Eq. (16). In this attempt one faces three kinds of problems. First, there is an uncertainty in the values of $a_i^{(DK\pi)}$ due to resonance contributions in fitted nonexotic D decays. Second, the values of $a_i^{(D\pi\pi)}$ may differ from those of $a_i^{(DK\pi)}$ which causes another uncertainty. In fact, a determination of $a_i^{(D\pi\pi)}$ from the corresponding Cabibbo suppressed decays (neglecting resonance contributions) gives very different results for a_2 compared with the $D\to K\pi$ case

$$a_1^{(D\pi\pi)}=1.05\left(\frac{0.7}{F_0^{D\pi}(m_\pi^2)}\right)\ ,\qquad a_2^{(D\pi\pi)}=-0.07\left(\frac{0.7}{F_0^{D\pi}(m_\pi^2)}\right)\ , \qquad (17)$$

where $F_0^{D\pi}(m_\pi^2) = 0.7$ [20] is used for normalization. This large deviation was blamed on inelastic hadronic rescattering [23].

Finally, a third uncertainty in evaluating $R_2(m_D)$ using (16) is due to the present experimental error in the ratio of form factors $F_0^{DK}(0)/F_0^{D\pi}(0)$. It was noted in [22] that the value of $R_2(m_D)$ is very sensitive to this ratio. In Table 1 we list the results of four experiments for which the average value is $F_0^{DK}(0)/F_0^{D\pi}(0) = 1.00 \pm 0.08$.

	Mark III [24]	CLEO [25]	CLEO [26]	E687 [27]
$\frac{F_0^{DK}(0)}{F_0^{D\pi}(0)}$	0.951 ± 0.214	1.054 ± 0.246	0.990 ± 0.230	1.000 ± 0.110

Table 1. Experimental results for the ratio of $D \to \pi(K)$ form factors at $q^2 = 0$. In quoting the numbers we used $|V_{cd}/V_{cs}| = 0.226$.

We conclude that it is difficult to evaluate $R_2(m_D)$ and to explain its experimental value in a reliable manner within the generalized factorization approach. It is not entirely impossible that the failure to account for this large SU(3) breaking is due to resonant contributions in other D decay processes which modify the extracted values of a_i. Assuming, for instance, $a_2^{(DK\pi)}/a_1^{(DK\pi)} = -0.6$ [7], $a_i^{(D\pi\pi)} = a_i^{(DK\pi)}$, $F_0^{DK}(0)/F_0^{D\pi}(0) = 1.1$, one finds using Eq. (16) the value $R_2(m_D) = 0.64$ consistent with (15). Still, a probable explanation for this failure is the presence of signficant nonfactorizable nonresonant contributions.

In view of the situation of R_2 at the D mass, one should be aware of the possible presence of uncalculable nonfactorizable SU(3) breaking terms at the B mass. We will disregard such terms for the rest of the discussion and study $R_1(m_B)$ and $R_2(m_B)$ in the generalized factorization approximation, keeping in mind that larger SU(3) breaking may be caused by nonfactorizable contributions.

In the factorization approximation one has

$$R_{1,2}(m_B) = \frac{a_{1,2}^{(BK\pi)}}{a_1^{(B\pi\pi)} + a_2^{(B\pi\pi)}} \frac{f_K}{f_\pi} \frac{F_0^{B\pi}(m_K^2)}{F_0^{B\pi}(m_\pi^2)} + \frac{a_{2,1}^{(BK\pi)}}{a_1^{(B\pi\pi)} + a_2^{(B\pi\pi)}} \frac{m_B^2 - m_K^2}{m_B^2 - m_\pi^2} \frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_\pi^2)} , \qquad (18)$$

where $R_2(m_B)$ is given by an expression analogous to (16). The parameters $a_i^{(BK\pi)}$ and $a_i^{(B\pi\pi)}$ cannot be determined direcly from experiments. The closest one can get empirically is to measure these parameters at a different scale, the scale of hadronic $b \to c$ decays. An analysis of measured rates for $B \to D^{(*)}\pi(\rho)$ and $B \to J/\psi K$, yields values [3, 28] $a_1^{BD\pi} \simeq 1$ and $a_2^{BD\pi} = 0.2 - 0.3$. A recent perturbative QCD calculation of $B \to \pi\pi$ decays [5], including nonfactorizable contributions due to hard gluon exchange, suggests that the corresponding value of the effective a_2 for two light pions could be even smaller, around $a_2^{(B\pi\pi)} = 0.1$. Actually, a_2 acquires a sizable complex phase. This calculation does not include nonfactorizable terms due to soft exchanges, which are argued to be power suppressed in the heavy quark limit. A precise calculation of these soft corrections is a challenging task. In our estimate below of the SU(3) breaking parameters $R_{1,2}$ we will use the range $a_2 = 0.1 - 0.3$, assuming for simplicity $a_i^{(BK\pi)} = a_i^{(B\pi\pi)}$. Note that in general a_i acquire complex phases and therefore R_i become complex. Neglecting complex phases has a small effect on our estimates.

Under these assumptions, it is convenient to introduce the sum and difference of R_1 and R_2, in which a dependence on $(a_1-a_2)/(a_1+a_2)$ is restricted to the difference

$$R_1+R_2 = \frac{f_K}{f_\pi}\frac{F_0^{B\pi}(m_K^2)}{F_0^{B\pi}(m_\pi^2)}+\frac{m_B^2-m_K^2}{m_B^2-m_\pi^2}\frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_\pi^2)}\ , \tag{19}$$

$$R_1-R_2 = \frac{a_1-a_2}{a_1+a_2}\left(\frac{f_K}{f_\pi}\frac{F_0^{B\pi}(m_K^2)}{F_0^{B\pi}(m_\pi^2)}-\frac{m_B^2-m_K^2}{m_B^2-m_\pi^2}\frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_\pi^2)}\right)\ . \tag{20}$$

The sum R_1+R_2 can be estimated more reliably than the difference R_1-R_2, since the former does not depend on the poorly known coefficients $a_{1,2}$.

Important ingredients entering the factorization expressions (18) are the hadronic form factors $F_0^{BP}(q^2)$, defined in the usual way [20]

$$\langle P(p_P)|\bar{b}\gamma_\mu q|B(p_B)\rangle = \left((p_B+p_P)_\mu-\frac{m_B^2-m_P^2}{q^2}q_\mu\right)F_1^{BP}(q^2)+\frac{m_B^2-m_P^2}{q^2}q_\mu F_0^{BP}(q^2)\ , \tag{21}$$

where $q=p_B-p_P$. The form factors $F_0^{B\pi(K)}(0)$ were computed in a variety of quark models [20, 29], light front model [30], MIT bag model [31], QCD sum rules [32, 33, 34] and lattice QCD [35]. The results obtained for these form factors at $q^2=0$ are presented in Table 2.

	BSW [20]	QCDSR [33]	LCSR [34]	RQM [29]	LFM [30]	BM [31]	Lattice QCD [35]
$F_0^{B\pi}(0)$	0.33	0.24	0.30 ± 0.04	0.37 ± 0.12	0.26	0.33	0.27 ± 0.11
$F_0^{BK}(0)$	0.38	0.25	0.35 ± 0.05	0.26 ± 0.08	0.34	–	–

Table 2. Theory predictions for semileptonic $B\to\pi(K)$ form factors at $q^2=0$.

The ratio of form factors $F_0^{B\pi}(m_K^2)/F_0^{B\pi}(m_\pi^2)$ is expected to differ from 1 by less than one percent; this difference will be neglected in the following discussion. Using the numerical values [20, 34] in Table 1 gives a typical value for the form factor ratio appearing in the second term of (18)

$$\frac{F_0^{BK}(m_\pi^2)}{F_0^{B\pi}(m_\pi^2)}=1.16\ . \tag{22}$$

It is hard to assign a theoretical uncertainty to this value, considering the large spread of model-predictions, some of which [29] even involve values smaller than one. The particular value (22) implies a near cancellation of the two terms in (20) [8], giving $R_1-R_2=0.06(a_1-a_2)/(a_1+a_2)=0.05\ (0.03)$, corresponding to $a_2=0.1\ (0.3)$. Together with the sum $R_1+R_2=2.37$, this predicts $R_1=1.21\ (1.20)$ and $R_2=1.16\ (1.17)$. Thus, with the particular choice (22), SU(3) breaking in R_2/R_1 is at most about 4%.

In view of the wide range of model-dependent results for $F_0^{BK}(0)/F_0^{B\pi}(0)$ (see Table 2), and in order to perhaps narrow this range, we propose an alternative calculation of this ratio, which is based on the measured ratio of corresponding form-factors in D decays, $F_0^{DK}(0)/F_0^{D\pi}(0)=1.00\pm0.08$. Semileptonic B and D decay form factors, at points of equal $\pi(K)$ energy in the rest frame of the decaying meson, are related by a heavy quark symmetry scaling law [36]

$$F_0^{B\pi}(q_*^2)=\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25}\sqrt{\frac{m_D}{m_B}}F_0^{D\pi}(0)\,,\quad F_0^{BK}(q_*^2)=\left(\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right)^{-6/25}\sqrt{\frac{m_D}{m_B}}F_0^{DK}(0)\ . \tag{23}$$

The momentum transfer for B form factors corresponding to $q^2 = 0$ in D decays is $q_*^2 = 18.0$ GeV^2, for K in the final state, and $q_*^2 = 17.6$ GeV^2 for π. Taking the double ratio of B and D form factors [37] cancels the leading $\mathcal{O}(1/m_Q)$ and $\mathcal{O}(m_s/\Lambda_{\chi SB})$ corrections to the scaling laws of the individual form factors

$$\frac{F_0^{BK}(q_*^2)/F_0^{B\pi}(q_*^2)}{F_0^{DK}(0)/F_0^{D\pi}(0)} = 1 + \mathcal{O}(m_s/m_c - m_s/m_b) \ . \tag{24}$$

We use this relation to predict the ratio of B form factors (22) in terms of the corresponding ratio for D decays. The extrapolation of the former from q_*^2 down to $q^2 = 0$ is made by assuming dominance by the 0^+ states $B_{0(s)}$ for which we take $m_{B_0} = 5.7 - 5.8$ GeV, $m_{B_{s0}} = 5.8 - 5.9$ GeV. This gives

$$\frac{F_0^{BK}(0)}{F_0^{B\pi}(0)} = (1.013 \pm 0.002)\frac{F_0^{BK}(q_*^2)}{F_0^{B\pi}(q_*^2)} \simeq 1.01 \pm 0.11 \ , \tag{25}$$

where we introduced an error of 7% associated with the $\mathcal{O}(m_s/m_c)$ term in (24) [37]. The rest of the uncertainty is due to the error in $F_0^{DK}(0)/F_0^{D\pi}(0)$. This uncertainty is expected to be reduced in future experiments of semileptonic D decays. The relation between ratios of form factors in D and B decays can be tested by measuring $B \to \pi \ell \nu$ and $B \to K \ell^+ \ell^-$.

The value (25) is somewhat lower than the result (22) taken from certain models. Inserting this value into the relations (19) and (20), we find $R_1 + R_2 = 2.22 \pm 0.11$ and $R_1 - R_2 = (0.21 \pm 0.11)(a_1 - a_2)/(a_1 + a_2)$. The central values yield $R_1 = 1.20$ (1.17) and $R_2 = 1.02$ (1.05) for $a_2 = 0.1$ (0.3). This implies very small SU(3) breaking in R_2 and larger SU(3) breaking in R_2/R_1, at a level of 15% (10%). This is significantly higher than the 4% effect estimated from Eq. (22). An even larger SU(3) breaking in R_2/R_1 is obtained in the factorization approximation for values of $F_0^{BK}(0)/F_0^{B\pi}(0)$ which are smaller than 1.

We conclude with an interesting observation. Our discussion of the large measured SU(3) breaking in hadronic D decays indicates the likely need for a significant nonfactorizable nonresonant contribution. Such effects may be smaller in B decays but ought to be considered with care. In spite of this warning, one may argue from rather simple grounds that in the generalized factorization approximation SU(3) breaking in $R_2(m_D)$ is expected to be much larger than in $R_2(m_B)$. Assuming universal values for a_i, separately for B and D decays, both $R_2(m_B)$ in Eq. (18) and $R_2(m_D)$ in Eq. (16) consist of two SU(3) breaking contributions weighed by $a_2/(a_1 + a_2)$ and $a_1/(a_1 + a_2)$. In B decays, where $a_2/a_1 \sim 0.1 - 0.3$, the dominant a_1 term involves SU(3) breaking given by $F_0^{BK}(0)/F_0^{B\pi}(0) - 1$ which is expected to be at a level of 10%. On the other hand, in D decays in which $a_2/a_1 \sim (-0.6) - (-0.4)$ is large and negative, the 22% SU(3) breaking of f_K/f_π in the a_2 term may be effectively roughly doubled by the destructive interference of this term with the a_1 term.

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